Intermediate Microeconomics Exercise Class 1

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Content

Basic Mathematics Knowledge

- Integral e.g. welfare economics
- Differential
  - Total differential
  - Implicit differentiation
  - Partial derivative
  - Common function forms
- Ordinary differential equation
- Optimization

Lagrange Multipliers

How to solve the following problem:

$$\min_x f(x) \textit{ subject to } g(x) \leq 0?$$

Introduce $\lambda \geq 0$ and we have $L = f(x) + \lambda g(x)$.
It holds that

$$\begin{aligned} \min_x f(x) &\geq \min_x \max_\lambda f(x) + \lambda g(x) \\ &\geq \max_\lambda \min_x f(x) + \lambda g(x). \end{aligned}$$

The first inequality holds since $g(x) \leq 0$ and the second inequality holds since that for any function $f(x, y)$, we have $\max_x \min_y f(x, y) \leq \min_y \max_x f(x, y)$.

Lagrange Multipliers Cont'd

There are two situations:

- If $g(x) < 0$ which we call non-binding, we have $\lambda = 0$;
- If $g(x) = 0$ which we call binding, we have $\lambda > 0$.

Therefore, we get the KKT condition that

$$\begin{cases} \frac{\partial L}{\partial x} = \nabla f(x) + \lambda \nabla g(x) = 0 \\ \lambda \geq 0 \\ g(x) \leq 0 \\ \lambda g(x) = 0. \end{cases}$$

Theorem

In mathematical optimization, the Karush–Kuhn–Tucker (KKT) conditions, also known as the Kuhn–Tucker conditions, are first derivative tests (sometimes called first-order necessary conditions) for a solution in nonlinear programming to be optimal, provided that some regularity conditions are satisfied.

Lagrange Multipliers Cont'd

- However, due to the characteristics of economics, most of problems we consider in this class are binding. So, we just need to consider the situation that $g(x) = 0$, which comes back to the tradition situation in your Advanced Mathematics class.
- Furthermore, you can explore Dual Method on your own.

Review of Basic Economics

- Why Economics? Scarcity!
- Opportunity Cost: highest valued
- Nominal variables VS real variables: USD/RMB=7
- Quantity demanded VS demand so as supply
- Inverse function

Elasticity

- What's elasticity?
- Own-price elasticity of demand: negative? Giffen goods
- $Elasticity = \frac{P/Q}{slope}$
- How to maximize revenue?
- Factors affecting elasticity
  - Substitutes
  - Market definition
  - Time
  - Expenditure share

Elasticity Cont'd

- Cross-price elasticity of demand
- Income elasticity of demand
- $Elasticity = \frac{\partial \ln f(x)}{\partial \ln x}$
- Point elasticity VS arc elasticity

Question 1

Suppose a commodity market is composed of two consumers A and B and two producers I and J. The demand functions of two consumers A, B for the commodity are $Q_A = 200 - 2P$, $Q_B = 150 - P$, and P is the price of the commodity. The supply functions of producers I and J are $Q_I = -100 + 2P$ and $Q_J = -150 + 3P$, respectively.

a) Find the market demand function and supply function of the commodity.

b) Find the equilibrium price and output.

c) When the market price is 50, What is the demand price elasticity of the market?

d) When the market price is 100, find the supply price elasticity of the market.

Question 2

For lunch, Ada prefers to eat soup and bread in fixed proportions. When she eats X pints of soup, she prefers to eat $\sqrt{X}$ ounces of bread. If she has X pints of soup and more than $\sqrt{X}$ ounces of bread, she eats all the soup along with $\sqrt{X}$ ounces of bread, and throws the extra bread away. If she has X pints of soup and fewer than $\sqrt{X}$ ounces of bread (say Y ounces), she eats all the bread along with Y^2 ounces of soup and throws the extra soup away.

a) Draw Ada's indifference curves between soup and bread.

b) Assume she spends all her income on soup and bread. Plot her income-consumption curve, her Engel curve for soup, and her Engel curve for bread.

c) Derive her demand function for the two goods. [Note that demand function is a function of prices and income].

Question 3

Most countries have civilians' medical insurance system. The insurance system in some countries, like Singapore, is operated through obligatory deposits. It means that every civilian will have a medical insurance account and the civilian should deposit some income into this account obligatorily. Consider a consumer in Singapore with income Y. He will spend C on consumption, S_1 on medical account, and S_2 on ordinary deposit. Suppose his utility function is $U(C, S_1, S_2) = C^\gamma S_1^\alpha S_2^\beta$. And he has the budget constrain: $C + S_1 + S_2 = Y$. Assume L is the lower limit value of S_1.
a) Derive the demand function of S_1 and S_2 when L is unrestrained.
b) Derive the demand function of S_1 and S_2 when L is restrained.

Question 4

A utility-maximizing consumer has an income $m > 0$, which he/she allocates between two goods (1 and 2). For each good, the consumer faces a constant price, p_1 and p_2, respectively. For each of the following utility function, derive the consumer's optimal demand $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.

a) $u(x_1, x_2) = \sqrt{x_1 x_2}$

b) $u(x_1, x_2) = \min\{x_1^2, x_2\}$

c) $u(x_1, x_2) = 2\sqrt{x_1} + x_2$

d) $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$

e) $u(x_1, x_2) = \begin{cases} x_1 x_2 & \text{if } x_1 \geq x_2 \\ x_1^2 & \text{if } x_1 < x_2 \end{cases}$

f) $u(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{if } 0 \leq x_1 \leq 20 \\ 20 + x_2 & \text{if } x_1 > 20 \end{cases}$

Thanks!