Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		2113105

A Fire Field Drone Solution: Modeling, Estimation and Prediction

Summary

Australia, especially Victoria, is plagued by wildfires these years. In order to react to fire more quickly, Victoria makes the decision to establish "Rapid Bushfire Response. In this article, we set up several models to help arrange drones' locations and determine the optimal numbers and mix of the two kinds of drones.

Considering the balance between safety and economic, we establish models to estimate optimal numbers of SSA drones and Radio Repeater Drones. Using extremum method, we draw a conclusion that purchasing 16 Radio Repeater drones seems optimal. And our calculation shows that the optimal number of SSA drones to purchase is 44. We use the Ant Colony Optimization to solve the optimized route of SSA drones and the interval for them to reach one point twice, and then determine the optimal number by establishing a model to associate the interval with fatality and injury rate and conducting cost-benefit analysis.

After that, we use time series data to construct static models that explain the relationship between fire frequency, temperature, precipitation and other variables. Furthermore, we identified the time trend of the average annual temperature in Australia and made predictions for the next ten years. At the same time, we used the Autoregressive Integrated Moving Average model (ARIMA) to predict the precipitation for the next ten years. Finally, we use the predicted values of explanatory variables to make conditional predictions on fire frequency. We estimate that the average fire frequency in the next decade will increase by 8.7% compared with that in the past decade, while the predicted value of that in year 2030 may rise to 21.7%.

Furthermore, with the help of Hill-climbing and GIS technology, we analyze how many Radio Repeater drones we need to deploy to guarantee that the whole area is covered by radio signals and where to deploy them.

We have tested the operability and robustness of the models by iterative experiments and analysis of time complexity. Finally, we analyze strengths and weaknesses of our model, conclude our results and develop a detailed budget for CFA.

Keywords: Drone,Bushfire,Ant Colony Optimization,Hill Climbing Algorithm,Autoregressive Integrated Moving Average model (ARIMA).

Contents

1 Introduction

1.1 Problem Background

A huge mountain fire broke out in Australia in 2019, causing huge losses to the whole country, especially in New South Wales and East Victoria. The government and firefighters were fighting the blaze as hard as they could. With the development of science and technology, Victoria's Country Fire Authority (CFA) determined to purchase some drones, including SSA drones (drones for surveillance and situational awareness) and Radio Repeater drones, for a proposed new division, "Rapid Bushfire Response". SSA drones can monitor and report data from wearable devices on front-line personnel, while Radio Repeater drones can increase the maximum communication distance between front-line personnel and the Emergency Operations Center (EOC). Both of them play an important role in ensuring the security of firefighters.

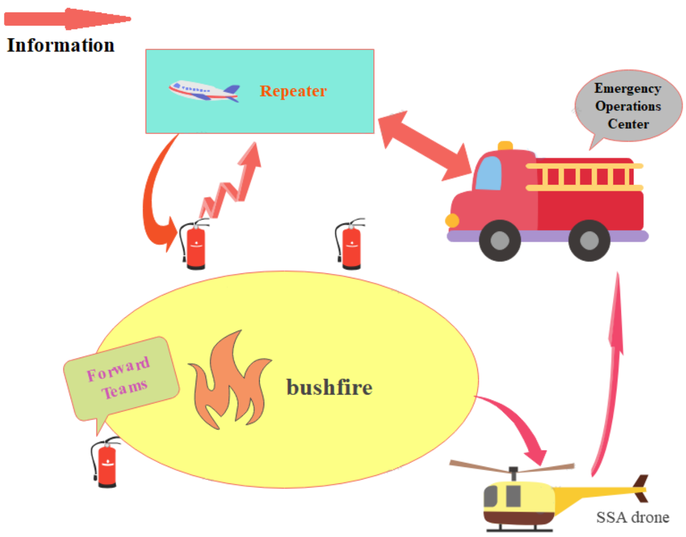


Figure 1: Relationships among the staff

1.2 Our work

However, it is not easy to achieve the balance between economy and safety. In this paper, we will:

1. take fires that have occurred in Australia as examples to build a model to determine the optimal number of two kinds of drones.
2. analyze factors that can affect the changes in the frequency of fires and predict it in the next decade based on historical data.
3. analyze the impact of changes in fire frequency, terrain factors, etc. on the drones, the model above and government budgets.

2 Preparation of the Models

2.1 Assumptions

1. The drone can be recycled (by charging to use next time), because the lifespan of firefighting drones in real life often exceeds the time of a fire. In addition, the two types of drones have the same basic attributes. Therefore, in order to simplify the cost calculation, it can be assumed that all drones have the same service life. Furthermore, it can be assumed that the drone can be recycled for a sufficiently long time.

2. The time for the drone to take off, land, and change direction is ignored. Since the time for a drone to take off, land and change direction is much less than the time of a flight, in order to simplify the calculation, this assumption can be made.

3. Regardless of the size of the fire, the time from the outbreak of the fire to the deployment of the drone is negligible. With the development of science and technology, the efficiency of detecting fire has become higher and higher. As for government agencies, the time required to deploy drones should be much less than the time it takes for drones to fly once, so the impact of this period on the fire situation can also be ignored.

4. The drone can be charged nearby. In consideration of the maximum working capacity of drones, the government is necessary and capable to quickly complete the layout of the drone chassis and enable all drones to transmit the information they collect back to EOC in a timely manner.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

3 The Models determining the numbers of drones

3.1 Model of SSA drones

3.1.1 Ant Colony optimization of cruising route

We assume that we should buy X SSA drones (hereinafter referred to as SSA), and when a fire occurs, the SSA will cruise and monitor the fire area at the speed of v.Let l be the distance that the drone cruises once and t be the time. Then, in order to minimize total cost, total cost of SSA(sumSSA) is determined as following:

$$sumSSA = X \times price + costSSA \tag{3.1}$$

where costSSA is the average cost caused by a fire.

Since the flying range of the SSA is $L_0 = 30km$, the cruising area of each SSA is a square with a side length of L_0. Assuming that the monitoring range of SSA is a square with side length $d_0 = 3km$, in order to allow SSA to monitor all fire areas in the cruise area, after dividing the fire area in the cruise area into square cells with side length d_0, SSA must pass through the center point of each cell.

Table 1: Notations

Symbol	Definition	Unit
X	the number of SSA drones to purchase	
Y	the number of Radio Repeater drones to purchase	
v	The maximum speed of an SSA	km/hr
L_0	The flying range of an SSA	km
t	The time for an SSA to cruise through all points	hr
T	The minimum value of t	hr
d_0	The diameter of the monitoring range of an SSA	km
p	the proportion of urban area	
S_i	area of a fire	km^2
p_i	probability of corresponding area	
R_u	a nominal range of a 5-watt radio in the urban	km
R_c	a nominal range of a 5-watt radio in the countryside	km
R	a nominal range of Radio Repeater drones	km
α	area correction factor	
$price$	unit price of a drone	
$cost$	total cost	AUD
h_i	elevation of a point	km
β	drone-safety factor	
T	The minimum time for an SSA to cruise through all points	hr
C	Compensation for one firefighter's fatality	AUD
γ	Possibility of firefighter fatality in bushfire task	
n	number of firefighters involved in certain area	
L	Expected loss caused by firefighter casualties	AUD
k	quantity of SSA put into use in certain area	

As the time interval t for it to reach the same point twice to monitor the information is smaller, firefighters can update the information faster, which means the safety of the firefighter is guaranteed better and the loss can be reduced at the same time. Since we assume that the speed of SSA is a constant $v = 72km/hr$, we need to find the minimum value of l first, and the minimum time T can be solved by the formula:

$$T = \frac{\min l}{v} \tag{3.2}$$

In fact, the problem of solving $\min l$ can be reduced to solving the shortest closed path that passes through the center of each small square exactly once. This is the famous TSP problem. We first estimate the fire area with a square with side length d in the Geometry Sketchpad software and mark the coordinates of each small square, and then use the Ant Colony Optimization to approximate $\min l$. Take the 2009 Beechworth fire as an example. We calculated five areas in the fire that need drones to cruise and the shortest path corresponding to each area. The result is shown in the figures and table below.

Note: For the purpose of drawing, the unit in the top picture is 3km, while the unit in the other five pictures is 0.05km.

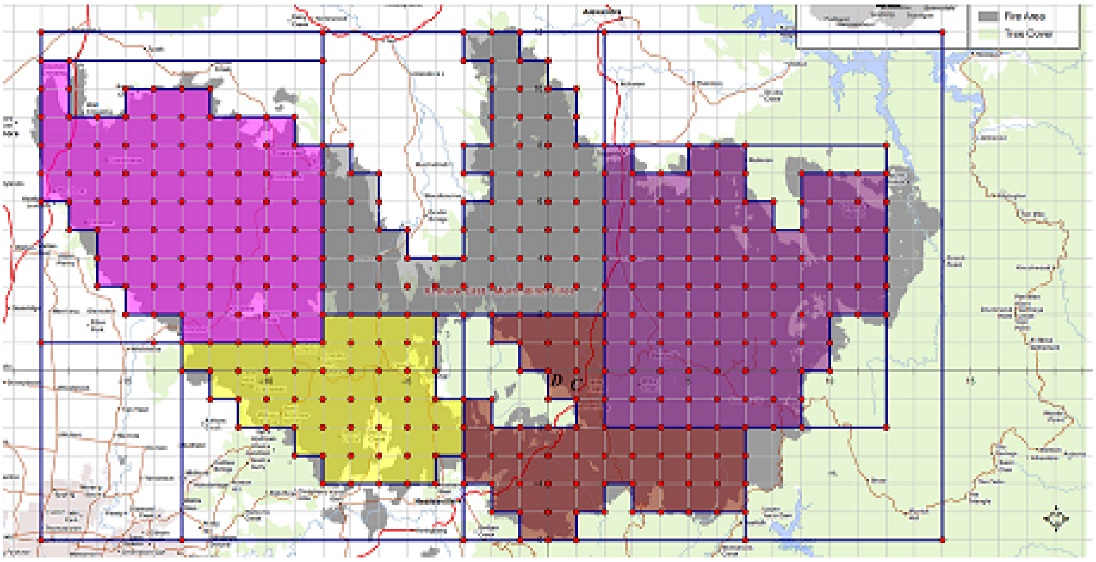


Figure 2: Five cruising area

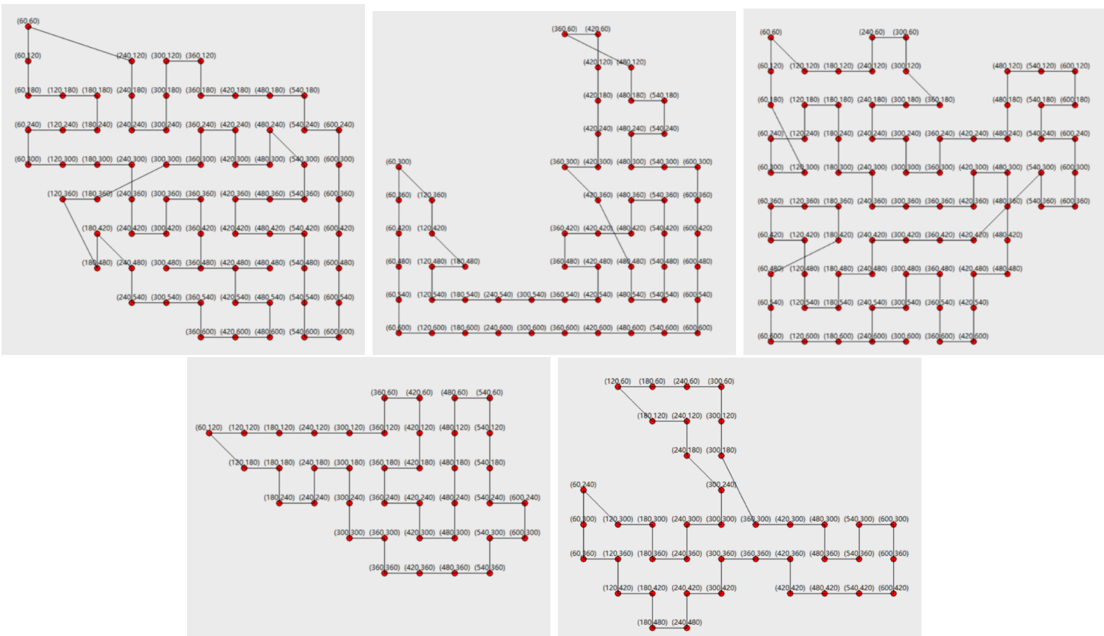


Figure 3: the shortest path corresponding to five areas

Table 2: Output

number	1	2	3	4	5
location	North West	North	North East	South West	South East
area(km^2)	639	513	720	351	369
costrepe ($0.05km$)	4708	2365	3643	5168	2609
T(h)	3.269	1.642	2.530	3.589	1.812

3.1.2 Cost-Benefit analysis and Optimization

The notations used in this subsection are listed in Table 1.

In this sections, we establish a model to estimate the benefit that each SSA drone could bring us and determine the optimal purchase quantity of SSA drone through cost-benefit analysis(that is, to compare the marginal cost and marginal return of each SSA drone).

Firefighters may encounter unexpected situations and accidents(surrounded by fire or hit by a fallen tree) and get injured or even lose their lives, which leads to huge loss in a bushfire.According to statistics, 7,373 firefighters were involved in the Australian fires from 2019 to 2020, and 9 of them lost their lives.[1]SSA drones monitor and report data from the wearable devices on the firefighters and promote situational awareness for those front line personnel in the complex fire environment.

The environmental conditions at various points in the fire are constantly changing.We assume that after a firefighter enters the fire field, the probability of an accident per unit time is $f(t)$,where t refers the time firefighters stay on the fire scene.When a firefighter enters the fire scene, he has complete information about his environment and the probability of an accident is zero. The environmental information possessed by firefighters becomes lagging with the passage of time, and the probability of encountering accidents increases linearly with time. Firefighters have actual combat experience and can talk to EOC via radio. Therefore, the probability has a maximum value of h. The time from the initial state to reaching the maximum value is T_s, that is

$$f(t) = \begin{cases} \frac{h}{T_s}t, & t < T_s \\ h, & otherwise \end{cases} \tag{3.3}$$

Suppose the total time of firefighting tasks is T_t, which meets the condition that

$$T_t >> T_s$$

then (as is shown in Figure 4.a)

$$\gamma = \int_0^{T_t} f(t)dt \approx h \cdot T_t \tag{3.4}$$

Whenever the SSA aircraft cruises past the firefighter's position, the firefighter's information is refreshed, and the accident probability $f(t)$ is refreshed to 0. After the aircraft leaves, the accident probability linearly increases until it reaches the maximum value h or be refreshed to the initial value 0 again by the next drone.Expectation loss L can be solved by the formula:

$$L = n\gamma C \tag{3.5}$$

To simplify the model, assume that

$$\frac{T}{2} < T_s < T$$

If we include the first SSA drone (as is shown in Figure 4.b), the fatality possibility γ_1 can be calculated by adding up shadow areas, which indicates that

$$\frac{\gamma_1}{\gamma} = \frac{ShadowArea}{TotalArea}$$

[1]AFAC NRSC numbers from Australia's largest deployment: https://www.afac.com.au/auxiliary/publications/newsletter/article/afac-nrsc-numbers-from-australia's-largest-deployment

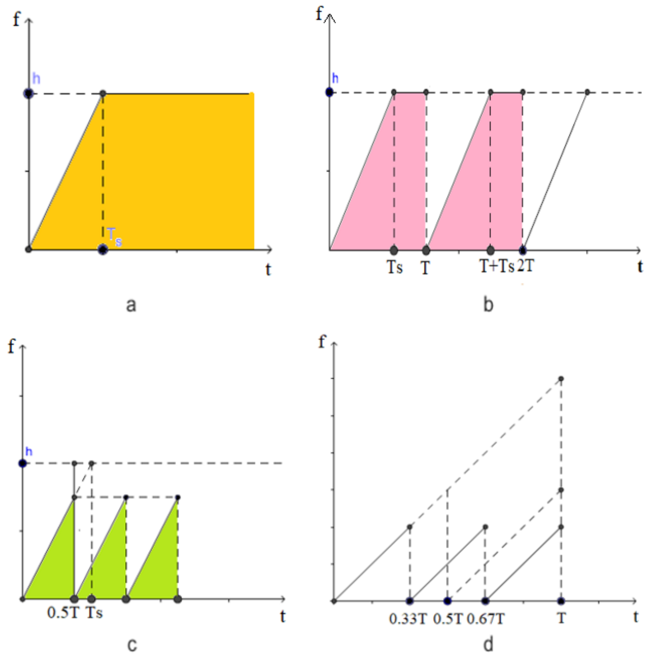


Figure 4: fatality rate and number of SSA drones

from which we can easily solve γ_1. Consider the rectangle $[0,T]\times[0,h]$:

$$\begin{aligned}\frac{\gamma_1}{\gamma} &= \frac{ShadowArea}{TotalArea}\\ &= 1-\frac{\frac{1}{2}hT_s}{hT}\end{aligned}$$

$$\Rightarrow \gamma_1 = \left(1-\frac{T_s}{2T}\right)\gamma \tag{3.6}$$

then, we can calculate the Marginal return of the first SSA, that is, the loss reduced by it

$$\begin{aligned}\Rightarrow \Delta L_1 &= L - L_1\\ &= n\gamma C - n\gamma_1 C\\ &= \frac{T_s}{2T}L\end{aligned} \tag{3.7}$$

Now we introduce a second SSA drone to our model.As is shown in Figure 4.c, consider the rectangle $[0,\frac{T}{2}]\times[0,h]$:

$$\begin{aligned}\frac{\gamma_2}{\gamma} &= \frac{ShadowArea}{TotalArea}\\ &= \frac{\frac{1}{2}\cdot(\frac{T}{2})^2\cdot(\frac{h}{T_s})}{\frac{T}{2}\cdot h}\end{aligned} \tag{3.8}$$

$$\Rightarrow \gamma_2 = \frac{T}{4T_s}\gamma, \quad L_2 = \frac{T}{4T_s}n\gamma C \tag{3.9}$$

$$
\begin{aligned}
\Rightarrow \Delta L_2 &= L_1 - L_2 \\
&= n\gamma_1 C - n\gamma_2 C \\
&= (1 - \frac{T_s}{2T} - \frac{T}{4T_s})L
\end{aligned} \tag{3.10}
$$

When $k \geq 3$,the analysis becomes easier and provides us a recurrence formula.As is shown in Figure 4.d, the shadow area change from two triangles, of which the area is $\frac{1}{2}$ of the big triangle, to three smaller triangles,of which the area is $\frac{1}{3}$ of the big triangle, which indicates:

$$
\gamma_3 = \frac{2}{3}\gamma_2, \quad L_3 = \frac{2}{3}L_2, \quad \Delta L_3 = \frac{1}{3}L_2
$$

Similarly,

$$
\gamma_{k+1} = \frac{k}{k+1}\gamma_k, \quad L_{k+1} = \frac{k}{k+1}L_k, \quad \Delta L_{k+1} = \frac{1}{k+1}L_k \tag{3.11}
$$

$$
\Rightarrow L_k = \prod_{j=3}^{k} \frac{j-1}{j} L_2 = \frac{2}{k}L_2, \quad \Delta L_{k+1} = \frac{2}{k(k+1)}L_2 = \frac{T}{2T_s}\frac{1}{k(k+1)} \cdot n\gamma C \tag{3.12}
$$

Substituting the formula of L_2 gives the formula of ΔL_{k+1}, which is the estimated marginal return of the $k+1^{th}$ SSA drone.It should be especially mentioned that, all "SSA drone" used in this section refers to the concept of "effective SSA drone".Considering that it takes each SSA drone 1.75 hours to recharge after working for 2.50 hours, 1 unit of "real SSA drone" equals to only $\frac{10}{17}$ unit of "effective SSA drone", which means the marginal cost of SSA drone that we discussed above is 17,000 AUD.

According to the compensation standard in Australia and other countries, we believe that $C = 2,000,000$ could be a reasonable estimation.Injury cases may come along with fatality cases, the incidence of which are always proportional.Although the number of injuries is two to three times the number of deaths, the average compensation of the former one is far smaller than the latter one, so we ignore them in this basic model to simplify our analysis. Fatality rate γ is estimated from data in the 2019-2020 bushfire in Australia. Considering the rare scale of this inferno, this may overestimate the average fatality rate, but we would rather overestimate than underestimate the fatality rate out of consideration of being cautious.We choose 2 hours as a rough estimation of T_s.The parameter of n depends on the scale of the bushifire.In the 2009 Beechworth bushfire involved 210 front line personnel[2], which means the density of firefighters is around $1/12km^2$.

Obviously, the marginal return function of SSA drone exhibits diminishing return to scale. To optimize our purchase plan, we should find the proper k such that ΔL_k equals to 17,000 AUD. After substituting parameters estimated above into equation (3.12), letting marginal return equals to marginal cost and converting the number of effective drones to the number of real drones at the rate of 1:1.7, we calculate that the optimal purchase quantity of SSA drone in this case is 25 (Rounded up;6,4,6,5,4 in five regions in Figure 2 respectively).

According to equation (3.12), id the fire area witnesses a N-fold increase, the expectation of n and T both increase by a factor of N times. Then, the optimal k must increase to Nk to keep the marginal return equal to marginal cost, which indicates the proportional relationship

[2]The Fires and the Fire-related Deaths,Parliament of Victoria. 2009 Victorian Bushfires royal Commission. iSBN 978-0-9807408-2-0. Published July

between fire area and expected optimal number of SSA drones. Our analysis of major bushfires in Victoria from 1851 to 2009 shows that the average fire area is $4390km^2$ (also mentioned in 3.2.2), while that of Beechworth bushfire is around $2592km^2$.According to the analysis above, we determine that the expected optimal number of SSA drones is 44.

3.2 Model of Radio Repeater drones

3.2.1 Details about the Model

The detail can be described as following. We assume that we should buy Y Radio Repeater drones (hereafter called Repeater) and there will be m big fires within their working years. And (S_i, p_i) shows the area of each fire and the probability it happens. So, in order to minimize total cost, total cost of Repeater (costrepe) is determined as following:

$$costrepe = Y \times price + \sum_i cost(S_i) \times p_i \times m \tag{3.13}$$

As for each fire, to simplify details about topography, we put mountains and cities in the same category where handheld radios are affected by humans activities or topography. And, as a general rule, cities and mountains with forests suffer more losses in the fire, so we record them as "Urban" both and we will analyze topography in detail using model afterwards. So, there is an area of $p \times S_i$ where a 5-watts radio has a nominal range of $R_u = 2km$, while $R_c = 5km$ in other area. And $R = 20km$ is the range of a repeater. As circle isn't tessellation polygon, we need α which is larger than 1 to correct the number. Besides the drones need recharging for $h_r = 1.75hr$ and the maximum flight time $h_w = 2.5hr$, and we assume that each fire is much longer than h_w. And the amount of Repeater we need is

$$x = \frac{\alpha \times S_i \times p}{\pi \times (R+R_u)^2} \times \frac{h_w + h_r}{h_w} + \frac{\alpha \times S_i \times (1-p)}{\pi \times (R+R_c)^2} \times \frac{h_w + h_r}{h_w} \tag{3.14}$$

We assume that there is only one fire at one time. So, if $Y \geq x$, we assume the extra $cost(S_i) = 0$. But if Y < x, we assume q Repeater is used in urban area while Y-q is used in the countryside. So when the cost is $lost_u$ per km in the urban and $lost_c$ in the country, and as perimeter of a certain shape is proportional to the area, we use $\sqrt{S}$ to simulate the perimeter. So,

$$cost_u(S_i, q) = \sqrt{(p \times S_i - q \times \pi \times (R+R_u)^2 \times \frac{h_w}{h_w + h_r})} \times lost_u \tag{3.15}$$

$$cost_c(S_i, q) = \sqrt{((1-p) \times S_i - (Y-q) \times \pi \times (R+R_c)^2 \times \frac{h_w}{h_w + h_r})} \times lost_c \tag{3.16}$$

$$cost(S_i, q) = cost_u(S_i, q) + cost_c(S_i, q) \tag{3.17}$$

So we minimize the cost of each fire, when q meet the condition

$$\frac{\partial cost(S_i, q)}{\partial q} = 0 \tag{3.18}$$

And the minimum $cost(S_i, q) = costmin(S_i)$. So

$$costrepe = Y \times price + \sum_i costmin(S_i) \times p_i \tag{3.19}$$

And the best Y meets the condition

$$\frac{\partial costrepe}{\partial Y} = 0 \tag{3.20}$$

Above all, we will use the actual data of Australia to determine the optimal numbers of Repeater for "Rapid Bushfire Response" for CFA in the following parts.

3.2.2 Application in Victoria's Country Fire Authority

We have analyzed fifteen major bushfires in Victoria from 1851 to 2009, and we estimate probability with frequency. The fires mentioned in the history have an average area of $4390km^2$. And we analyze the basic situation of Victoria and make some assumptions in Table 3.

Table 3: Assumption

Variable	Assumption
m	12
p	0.3
α	$\frac{4}{\pi} \approx 1.273$
$price$	10000 AUD
$lost_u$	1500 AUD/km
$lost_c$	750 AUD/km

Using a C code, we apply the model above and traverse Y from 0 to 30 on Victoria, and partial results are as follows. Therefore, we can draw a conclusion that we recommend to purchase sixteen Radio Repeater drones so as to maintain the best balance capability and safety with economics.

Table 4: Output

number	12	13	14	15	16	17
costrepe (AUD)	221609.7	221395.3	219627.2	214922.8	199524.1	202936.8
number	18	19	20	21	22	23
costrepe (AUD)	204647.8	201410.2	200000	210000	220000	230000

4 Prediction of fire frequency and bushfire possibility

NASA's FIRMS provided data on all fires in Australia from 2000 to 2020. We assume that the frequency of forest fires and the probability of extreme fires are proportional to the frequency of these fires. We estimate how the frequency of fires in Australia from 2020 to 2030 will change in three steps: (1) Estimate which factors significantly affect the frequency of fires; (2) Estimate the changes in these influencing factors in the next ten years; (3) Estimate the frequency of fires in the next ten years.

4.1 Estimate factors that affect fire frequency

It is generally believed that climate change and human activities are the two major factors affecting the frequency of fires. Here, we select Australia's annual average temperature, annual rainfall, total population and population growth rate as explanatory variables. At the same time, considering that the data of the explained variable (fire frequency) is obtained through satellite observations, the daytime cloud may affect the visibility of satellite observations and thus the explained variable. Preliminary model testing found that the effect of population total or population growth rate on the explained variables is extremely insignificant, so we will exclude them in the following analysis.

We set a static model as follows,

$$lfrcnt_t = \beta_0 + \beta_1 mntemp_t + \beta_2 mntemp_t^2 + \beta_3 rainfl_t + \beta_4 cloud_t + u_t \tag{4.1}$$

where $lfrcnt$ refers to the natural logarithm of fire frequency in year t,$mntemp$ refers to the average annual temperature of the year, $rainfl$ refers to the total rainfall of the year, and $cloud$ refers to the annual average daytime cloud. We tested all variables and find that there is no obvious trend between the explanatory variables and the explained variables in the model except for $mntemp$ so as to exclude the spurious regression problem.There is no complete collinearity between the explanatory variables. Due to the potential heteroscedasticity problem, we give both standard errors and heteroscedasticity robust standard errors. The regression results are as follows:

$$\begin{array}{llllll}
\widehat{lfrcnt_t} = 20.39 & -1.600 mntemp & +0.8870 mntemp^2 & -3.135 cloud & +0.004924 rainfl \\
\quad (1.678) & (0.4433) & (0.2942) & (0.7970) & (0.001953) \\
\quad [1.434] & [0.5998] & [0.3580] & [0.6987] & [0.001932]
\end{array}$$

$$n = 19 \qquad R^2 = 0.6736 \qquad \bar{R}^2 = 0.5804$$

All explanatory variables are significant at the 5% level. Contrary to intuition,the coefficient of $rainfl$ is positive, which may be caused by the multicollinearity between $cloud$ and $rainfl$.This anti-intuitive regression coefficient might also indicate the relationship between rainfall and lightning, which could potentially trigger a bush fire.

4.2 Predict the explanatory variables

In this section, we try to estimate how explanatory variables will change in the next decade. The explanatory variable of mean temperature shows a strong trend. We try to use time series regression with time term to identify this trend and predict the future average temperature.According to the test results in the table below, we decided to include the quadratic term of time into the model.[3]

Model (2) has a larger Adjusted R-square,and indicates an accelerating trend of global warming.Using model (2), we estimate the annual mean temperature in the next decade.In 2030, mean temperature is expected to be 1.463, which is consistent with expectation of IPCC.[4]

[3] data source: Australian Government,Bureau of Meteorology

[4] IPCC, 2018: Global Warming of 1.5°C.An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty Masson-Delmotte, V., P. Zhai, H.-O. Pörtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I. Gomis, E. Lonnoy, T. Maycock, M. Tignor, and T. Waterfield (eds.)]. In Press.

Table 5: Time trend of mean temperature:1920-2020

	(1)	(2)
	mntemp	*mntemp*
year	0.0149***	-0.72577***
	(0.00119)	(0.165)
$year^2$		0.000188***
		(0.0000418)
_cons	-29.38***	700.0***
	(2.345)	(162.1)
N	101	101
R^2	0.614	0.680
adj. R^2	0.610	0.673

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Rainfl,on the contrary, shows no significant trend.We use ARIMA model to predict the rainfall in the future. DF test shows that rainfall data has better stationarity, so we set term d to 0.ARMA model is set as follows:

$$rainfl_t = \mu + \sum_{i=1}^{p} \gamma_i rainfl_{t-i} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{4.2}$$

After testing the autocorrelation and partial autocorrelation of rainfall, we select model ARMA(0,1).According to the ARMA(0,1) model, there is little autocorrelation between $rainfl_t$ and $rainfl_{t-1}$. As a result, the mathematics expectation of rain remains relatively stable in the next decade, estimated 461.1.

4.3 Predict fire frequency

We have predicted the future $rainfl$ and $mntemp$, but note the $cloud$ yet. On the one hand, $cloud$ and$rainfl$ have a highly collinear relationship, which can be simulated with a moving average model, and its expected value remains relatively stable. More importantly, what we ultimately hope to get is the real fire frequency, not the fire frequency observed by satellites. We believe that the negative correlation between cloud and fire frequency is mainly caused by clouds' obstructing effect on satellite observations. Therefore, we will only use the partial effect of $mntemp$ and $rainfl$ after controlling the $cloud$ (that is, controlling the observation error) to predict how the natural logarithm of the fire frequency after controlling the observation factor will change in the next decade, and then use it to estimate the percentage of fire frequency change in the next ten years.

To compare the fire frequency(also, the bushfire possibility which is assumed to be proportional with fire frequency) in the next decade with that in the past ten years, we estimate the average $mntemp$, average $mntemp^2$ and average $rainfl$ from 2011 to 2020 and predict those data from 2021 to 2030.

According to the model we estimated above, if we control the value of $cloud$,then:

$$\Delta\overline{lfrcnt} = -1.600\Delta\overline{mntemp} + 0.8870\Delta\overline{mntemp}^2 + 0.004924\Delta\overline{rainfl}$$

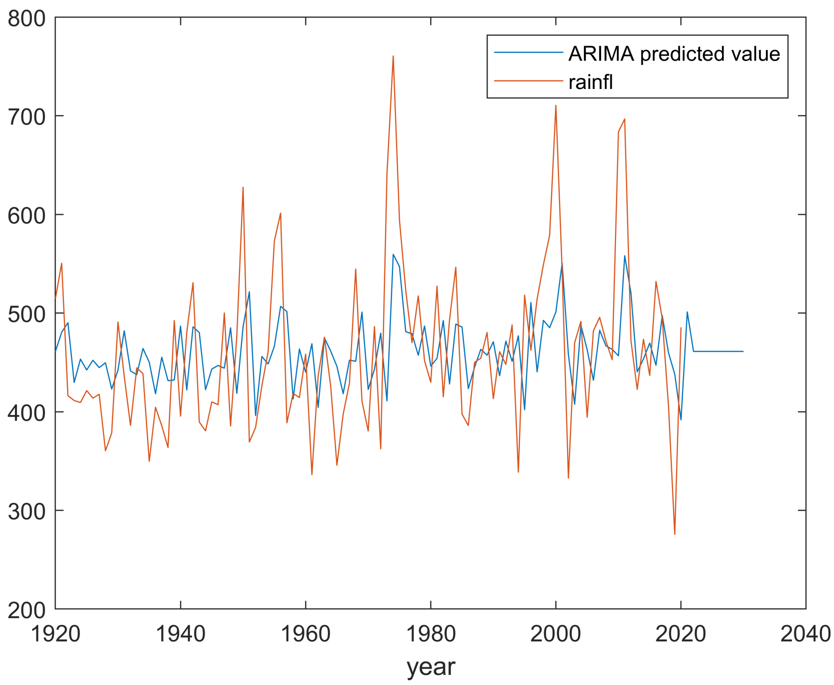


Figure 5: Result of ARIMA

Table 6: Estimated parameters in two decades

period	$\overline{mntemp}$	$\overline{rainfl}$
2011-2020	0.939	469.48
2021-2030	1.2797	461.08
delta	0.3407	-8.4

and

$$\%ChangeFirecnt = (e^{\Delta \overline{lfrcnt}} - 1) \cdot 100\%$$

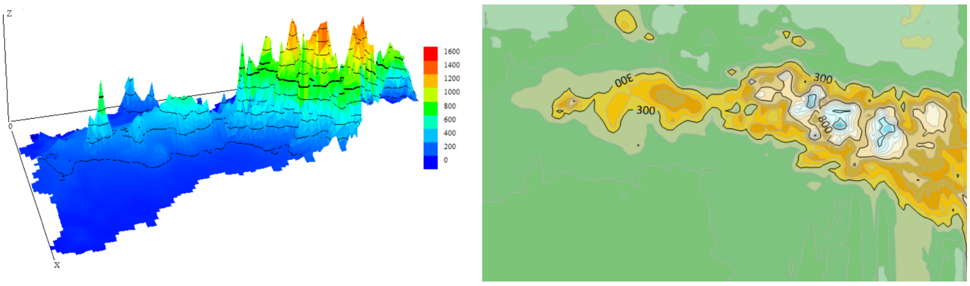
Substituting the data estimated above, we predict that the average annual fire frequency(as well as bushfire possibility) in the next decade will increase by 8.77%.

An expected 8.77% increase in the fire frequency can affect the result determined in the previous model in two ways:(1)Frequency and possibility of bushfire of large scale may increase proportionally, that is to say, the parameter m in the model where the optimal quantity of radio repeater drones is determined will increase by 8.77%, which causes the optimal number of repeater drones to increase by 1 and an extra equipment cost of $10,000(AUD).In 2030, if fire frequency increase by 21.7% as predicted above, the optimal number will change to 24, which costs another $80,000(AUD).(2)Increase in the frequency of fire may imply the increase in the average bushfire scale and a higher possibility of potential huge bushfire, which indicates a increase in optimal quantity of both SSA and radio repeater drones.

5 The Model determining the locations of Repeaters

5.1 Details about the Model

In this part, we consider a different model from the models before. We assume that we have enough Repeaters for each fire and this model trys to finding the best locations of hovering VHF/UHF radio-repeater drones.



(a) 3D topographic map of Victoria (b) Plain topographic map of Victoria

Figure 6: Topographic map of Victoria

Using Figure above, we can know the altitude of each point in the area on fire. So, we divide the area into a lot of small areas which are regards a set of points. And our drones can hover beyond each point. Knowing the altitude (h_i) of each point, in order to keep our drones safe, the flight altitude of each point must be larger than $H_i = \beta \times h_i$. And, we use a boolean arguments $flag$ which can only be 0 or 1 matching with R_c or R_u. The radius are regarded as on the ground, because $R \gg h_i$. And the existence of finite circles which cover an area is guaranteed by the Vitali covering lemma. So the valid radius

$$R_i = \sqrt{(R + \phi(flag_i))^2 - H_i^2)} \tag{5.1}$$

And $\phi(x)$ is as following:

$$\phi(x) = \begin{cases} 5 & x = 0 \\ 2 & x = 1 \end{cases}$$

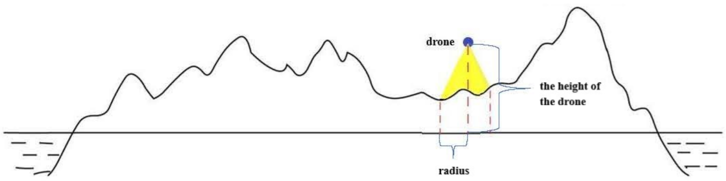


Figure 7: Sketch map of the hovering location

So with the assumption above, we use Hill Climbing Algorithm to optimizing the locations of hovering Repeaters. We assume that the area on fire is divided into max parts. The pseudo code is as following:

Pseudo Code:

```
while(n<max)
simulate every situation with n repeaters
```

```
if(the coverage radius of all drones cover all points)
the locations of these n drones are what we need
else next situation
then next n
```

As the more drones we use, the more area we can cover. So, this Hill Climbing Algorithm is the optimal solution in each case with a certain area on fire.

5.2 Time complexity for fires of different sizes on different terrains

When the size of area on fire increases, traversal is an NPC problem, so we use Hill Climbing Algorithm to find optimal solution with polynomial time complexity nearly. Unlike the method enumerating possible solution vectors or adapting Greedy Algorithm, it balances velocity and accuracy. Enumeration has a time complexity about at least O(2^n), and it has exact solution, while Greedy Algorithm has polynomial time complexity with a local optimal solution which may not be global optimal solution.

5.3 A example of fire with certain size and terrain

- We use the 3D topographic map of Victoria and image a relatively small fire about $1296km^2$. To simplify the model, we assume there are one hundred uniform points in the square. Because we are interested in the sizes and the terrains, the points represent Victoria whose altitude varies from sea level to more than 1500 meters as Mt.Bogong is 1986 meters high.
- The result is as following:

Table 7: Basic Information about Three Experiments with Different Algorithms

Algorithm	Time	Answer
Greedy Algorithm	0.101 second	min:3
Hill Climbing	0.111 second	min:2
Enumeration	When the model has 36 points, it spends 81 seconds to get the answer, our personal computer has no capacity of getting the answer when the number of points is one hundred.	When the number of points is less than 36, the answer is right. So, we have reasons to believe that the algorithm is correct but it need plenty of time.

So the minimum in this case with certain size and terrain is two and the location of hovering is shown in Figure 8. And when facing different sizes and terrains, we can just use ”*question_three.cpp*” in the appendix to gain an optimal arrangement of drones in period time which can be accepted.

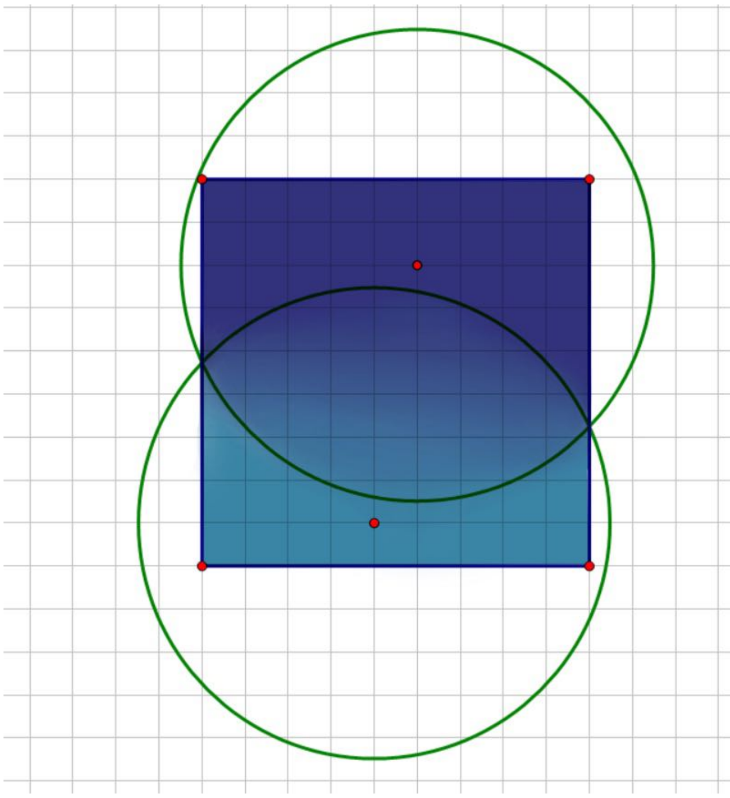


Figure 8: Locations of Repeaters optimally

6 Strengths and Weaknesses

6.1 Strengths

- Ant colony optimization(ACO) is self-organized and robust. Compared with similar algorithms, the solution of ACO is less influenced by initialization.
- The mechanism of the positive feedback of ACO improve the efficiency while guaranteeing the precision of the solution.
- The Hill Climbing Algorithm has a wonderful balance with time complexity with accuracy. It can calculate the optimal locations of hovering quickly enough to help EOC work out a plan.
- Although we use a simple area as an example showing how to arrange the locations of hovering, our c++ code has great portability. When facing actual bushfire, EOC could just input precise location information to gain the location arrangement.
- The formula of marginal return of SSA drone is inversely proportional to the square of k, which means the optimal k determined in the model is relatively not sensitive to changes in parameters (The elasticity of k to any other parameter is around 0.5, significantly lower than 1).
- The overall explanatory power of the fire frequency prediction model is relatively high and the coefficients in the regression model has good significant levels while exhibiting heteroscedasticity robustness.

6.2 Weaknesses

- The convergence rate of ACO is relatively slow, especially in the beginning. Consequently, in order to get a more precise solution, we need to increase the iterations.
- The Hill Algorithm may be degraded to exponential time complexity which is nearly impossible for us to gain the optimal answer.
- The algorithm we use can have a further optimization. We can use an adjacency matrix or Edge Table to store the relationship between point sets so that we can reduce the time spending on finding which points are in the communication range of a certain VHF/UHF radio-repeater drones.
- The estimation of parameters in the cost-benefit analysis model lacks enough data support
- The prediction model doesn't include human activities' impact on fire frequency.

7 Conclusion

Considering the balance between safety and economic, we establish models to estimate optimal numbers of SSA drones and Radio Repeater Drones. After that, we predict the expected fire frequency according to historical data. Furthermore, with the help of Hill-climbing and GIS technology, we analyze how many Radio Repeater drones we need to deploy to guarantee that the whole area is covered by radio signals and where to deploy them. Finally, we tested the operability and robustness of the models by iterative experiments and analysis of time complexity, ending up with the discussion of the strengths and weaknesses of models.

Budget Request

Budget Summary

WileE–15.2X Hybrid Drones. $600,000(AUD) will be used to purchase 60 drones for bushfire suppression

Budget Annotation

SSA Drones

Drones for surveillance and situational awareness (SSA): $440,000 for 44 drones

In the complex environment of bushfire, drones for surveillance and situational awareness can monitor the evolving situation, collect and report data from the wearable devices on the front line personnel.SSA drones help the Emergency Operations Center(EOC) direct firefighters more effective and reduce the risk by providing firefighters information of their situation.As is estimated by the consultant team, the fatality and injury rate in the 2009 Beechworth bushfire would decrease by up to around 86.4% if 25 SSA drones were put into use, which means the loss might be reduced by $477,615(AUD) while only costing $250,000 (for 25 drones). Considering that average scale of bushfire in Victoria from 1851 to 2009 is $4390km^2$(in comparison with that of around $2592km^2$ in the Beechworth bushfire), the consultant team determine that the optimal number of SSA drones is 44.

Radio Repeater Carrier Drones

Radio repeater carrier drones: $160,000(AUD) for 16 drones

Radio Repeater drones are so significant since they guarantee the communication between EOC with forward teams. Using VHF&UHF, repeaters can help extend the communication radius to more than 20 kilometers, while a 5-watt radio that "Boots-on-the-ground" Forward Teams equip with only has a nominal range of 5 km over flat, unobstructed ground or 2 km in an urban area. To estimate the amount of Repeaters that satisfies with Victoria State Government, our consultant team use Hill Climbing Algorithm and it can be optimized using Dynamic Programming. As a result, in order to minimize the cost including safety cost and economic cost, the consultant team recommend you, government to purchase 16 Radio Repeater drones. Costing $ 160,000(AUD), the drones can meet most bushfire situations and perform well though facing extreme fire.

Equipment cost increase over the next decade

Considering the increasing probability of extreme fire, our consultant team predict that the frequency of bushfire will increase at an average annual growth rate of 8.7% over the next decade. So, unfortunately, the cost on drones used for fire flights will increase continuously. But anyway, purchasing optimal amounts of SSA drones and Radio Repeater drones seems the best choice to ensure the safety of people's lives and property and reduce the loss in the fire.

Conclusion

Overall the consultant team recommends to purchase 44 SSA drones and 16 Radio Repeater drones with a budget of $600,000(AUD).

Table 8: Purchasing List

type	amount	cost(AUD)
SSA drones	44	440,000
Radio Repeater drones	16	160,000
Sum		600,000

Victoria Country Fire Authority (CFA)

References

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[2] Ashe, Brian, Kevin John McAneney, and A. J. Pitman. "Total cost of fire in Australia." Journal of Risk Research 12.2 (2009): 121-136.

[3] AFAC NRSC numbers from Australia's largest deployment: `https://www.afac.com.au/auxiliary/publications/newsletter/article/afac-nrsc-numbers-from-australia's-largest-deployment`

[4] The Fires and the Fire-related Deaths,Parliament of Victoria. 2009 Victorian Bushfires royal Commission. iSBN 978-0-9807408-2-0. Published July

Appendix A: Program Codes

Here are the program codes we used in our research.

question_one.py

```
# Python code Ant Colony Optimization
import random
import copy
import time
import sys
import math
import tkinter
import threading
from functools import reduce
(ALPHA, BETA, RHO, Q) = ( , , , )
(cc_n, ant_n) = ( , )
dd_x = [ ]
dd_y = [ ]
dd_g = [ [0.0 for col in range(cc_n)] for raw in range(cc_n)]
pp_g = [ [1.0 for col in range(cc_n)] for raw in range(cc_n)]
class Ant(object):
def __init__(self,ID):
self.ID = ID
self.__clean_data()
def __clean_data(self):
self.path = []
self.total_dd = 0.0
self.move_count = 0
self.current_cc = -1
self.open_table_cc = [True for i in range(cc_n)]
cc_index = random.randint(0,cc_n-1)
self.current_cc = cc_index
self.path.append(cc_index)
self.open_table_cc[cc_index] = False
self.move_count = 1
def __choice_next_cc(self):
next_cc = -1
select_ccs_prob = [0.0 for i in range(cc_n)]
total_prob = 0.0
for i in range(cc_n):
if self.open_table_cc[i]:
try :
select_ccs_prob[i]=pow(pp_g[self.current_cc][i],
ALPHA)*pow((1.0/dd_g[self.current_cc][i]),BETA)
total_prob += select_ccs_prob[i]
except ZeroDivisionError as e:
print ('Ant ID: {ID}, current city: {current},
target city: {target}'.format(ID = self.ID,
current = self.current_cc, target = i))
sys.exit(1)
if total_prob > 0.0:
temp_prob = random.uniform(0.0, total_prob)
for i in range(cc_n):
```

```
if self.open_table_cc[i]:
temp_prob -= select_ccs_prob[i]
if temp_prob < 0.0:
next_cc = i
break
if (next_cc == -1):
next_cc = random.randint(0, cc_n - 1)
while ((self.open_table_cc[next_cc]) == False):
next_cc = random.randint(0, cc_n - 1)
return next_cc
def __cal_total_dd(self):
temp_dd = 0.0
for i in range(1, cc_n):
start, end = self.path[i], self.path[i-1]
temp_dd += dd_g[start][end]
end = self.path[0]
temp_dd += dd_g[start][end]
self.total_dd = temp_dd
def __move(self, next_cc):
self.path.append(next_cc)
self.open_table_cc[next_cc] = False
self.total_dd += dd_g[self.current_cc][next_cc]
self.current_cc = next_cc
self.move_count += 1
def search_path(self):
self.__clean_data()
while self.move_count < cc_n:
next_cc =  self.__choice_next_cc()
self.__move(next_cc)
self.__cal_total_dd()
class TSP(object):
def __init__(self, root, width = 800, height = 600, n = cc_n):
self.root = root
self.width = width
self.height = height
self.n = n
self.canvas = tkinter.Canvas(root,width = self.width,height
= self.height,
bg = "#EBEBEB",xscrollincrement = 1,
yscrollincrement = 1)
self.canvas.pack(expand = tkinter.YES, fill = tkinter.BOTH)
self.title("n:initialize e:start s:end q:quit")
self.__r = 5
self.__lock = threading.RLock()
self.__bindEvents()
self.new()
for i in range(cc_n):
for j in range(cc_n):
temp_dd = pow((dd_x[i] - dd_x[j]), 2) + pow((dd_y[i]
- dd_y[j]), 2)
temp_dd = pow(temp_dd, 0.5)
dd_g[i][j] =float(int(temp_dd + 0.5))
def __bindEvents(self):
```

```
self.root.bind("q", self.quite)
self.root.bind("n", self.new)
self.root.bind("e", self.search_path)
self.root.bind("s", self.stop)
def title(self, s):self.root.title(s)
def new(self, evt = None):
self.__lock.acquire()
self.__running = False
self.__lock.release()
self.clear()
self.nodes = []
self.nodes2 = []
for i in range(len(dd_x)):
x = dd_x[i]
y = dd_y[i]
self.nodes.append((x, y))
node = self.canvas.create_oval(x - self.__r,y - self.__r,
x + self.__r, y + self.__r,
fill = "#ff0000",outline = "#000000",
tags = "node",)
self.nodes2.append(node)
self.canvas.create_text(x,y-10,text = '('+str(x)+','
+str(y)+')',fill = 'black')
for i in range(cc_n):
for j in range(cc_n):pp_g[i][j] = 1.0
self.ants = [Ant(ID) for ID in range(ant_n)]
self.best_ant = Ant(-1)
self.best_ant.total_dd = 1 << 31
self.iter = 1
def line(self, order):
self.canvas.delete("line")
def line2(i1, i2):
p1, p2 = self.nodes[i1], self.nodes[i2]
self.canvas.create_line(p1,p2,fill="#000000",tags="line")
return i2
reduce(line2, order, order[-1])
def clear(self):
for item in self.canvas.find_all():self.canvas.delete(item)
def quite(self, evt):
self.__lock.acquire()
self.__running = False
self.__lock.release()
self.root.destroy()
print (u"\n end...")
sys.exit()
def stop(self, evt):
self.__lock.acquire()
self.__running = False
self.__lock.release()
def search_path(self, evt = None):
self.__lock.acquire()
self.__running = True
self.__lock.release()
```

```
while self.__running:
for ant in self.ants:
ant.search_path()
if ant.total_dd < self.best_ant.total_dd:
self.best_ant = copy.deepcopy(ant)
self.__update_pp_g()
print (u"number:",self.iter,u"min:",
int(self.best_ant.total_dd))
self.line(self.best_ant.path)
self.title("n:initialize e:start s:stop q:quit number:
%d" % self.iter)
self.canvas.update()
self.iter += 1
def __update_pp_g(self):
temp_pp=[[0.0 for col in range(cc_n)] for raw in range(cc_n)]
for ant in self.ants:
for i in range(1,cc_n):
start, end = ant.path[i-1], ant.path[i]
temp_pp[start][end] += Q / ant.total_dd
temp_pp[end][start] = temp_pp[start][end]
for i in range(cc_n):
for j in range(cc_n):pp_g[i][j] = pp_g[i][j] * RHO +
temp_pp[i][j]
def mainloop(self):self.root.mainloop()
if __name__ == '__main__':
TSP(tkinter.Tk()).mainloop()
```

question_three.cpp

```
// C++ code Hill Climbing
#include <iostream>
#include<stdio.h>
#include<math.h>
struct point{
int x,y,flag,height,drone,use,cover;
double rad;};
using namespace std;
int func(int fir,int n);
int judge(void);
void clean(void);
void sign(int i,int j);
const int num=10;     //determined by the size of the fire
struct point aus[num][num];
int main(){
//data of the terrain in this case
freopen("out.txt","r",stdin);
double bevel;
for(int i=0;i<num;++i)
for(int j=0;j<num;++j){
cin>>aus[i][j].x>>aus[i][j].y>>aus[i][j].flag
>>aus[i][j].height;
aus[i][j].drone=5*aus[i][j].height;
if(aus[i][j].flag==0)bevel=25;
```

```
else bevel=22;
aus[i][j].rad=sqrt(bevel*bevel-aus[i][j].drone*
aus[i][j].drone*1.0/1000.0/1000.0);
aus[i][j].use=aus[i][j].cover=0;}
for(int i=1;i<=num*num;++i)if(func(0,i)){
cout<<"min:"<<i<<endl;
return 0;}
cout<<"no solution"<<endl;
return 0;
}
int func(int fir,int n){
if(n==0){
if(judge()){
cout<<"location:"<<endl;
for(int i=0;i<num;++i)
for(int j=0;j<num;++j)
if(aus[i][j].use==1)cout<<"x:"<<aus[i][j].x
<<" "<<"y:"<<aus[i][j].y<<" "<<endl;
cout<<endl;
return 1;}
else {
clean();
return 0;}
}
if(fir==num*num)return 0;
for(int i=fir;i<num*num;++i){
int x=i/10,y=i-10*x;
aus[x][y].use=1;
int ans=func(i+1,n-1);
aus[x][y].use=0;
if(ans==1)return 1;}
return 0;
}
void clean(void){
for(int i=0;i<num;++i){
for(int j=0;j<num;++j)aus[i][j].cover=0;}
return;
}
int judge(void){
for(int i=0;i<num;++i)
for(int j=0;j<num;++j)if(aus[i][j].use==1)sign(i,j);
for(int i=0;i<num;++i)
for(int j=0;j<num;++j)if(aus[i][j].cover==0)return 0;
return 1;
}
void sign(int i,int j){
for(int x=0;x<num;x++)
for(int y=0;y<num;++y)
if((aus[x][y].x-aus[i][j].x)*(aus[x][y].x-aus[i][j].x)+
(aus[x][y].y-aus[i][j].y)*(aus[x][y].y-aus[i][j].y)<
aus[i][j].rad*aus[i][j].rad)aus[x][y].cover=1;
return;
}
```